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EXAMPLE OF AN EXACT SOLUTION OF THE PROBLEM OF THE DISTRIBUTION OF AN IONIZED IMPURITY IN THE SURFACE REGION OF A SEMICONDUCTOR

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An exact solution of the nonlinear problem of determining the electrostatic potential and the distribution profile of an ionized impurity in the surface region of a semiconductor is constructed.

Introduction. We consider the one-dimensional case of the distribution of an ionized impurity in a n-type semiconductor under steady-state conditions in an external electric field. The distribution of the electrostatic potential Ψ as a function of the x coordinate reckoned from the surface of the semiconductor is described by the Poisson equation

$$\frac{d^2\Psi}{dx^2} = -\frac{q}{\varepsilon_0\,\varepsilon_s}[N - n], \tag{1}$$

where $N(x)$ is the distribution profile of the ionized impurity. The electron density n under conditions close to thermodynamic equilibrium is expressed in terms of the electrochemical potential φ_0 [1]:

$$n = n_i \exp(\beta(\Psi - \varphi_0)), \qquad \beta = q/(kT). \tag{2}$$

Here n_i is the density of carriers in the semiconductor itself, q is the electron charge ($q = 1.6 \cdot 10^{-19}$ C), T is the temperature ($T = 300$ K), k is the Boltzmann constant ($k = 1.38 \cdot 10^{-23}$ J/K), ε_0 is the permittivity of vacuum ($\varepsilon_0 = 8.86 \cdot 10^{-14}$ F/cm), and ε_s is the dielectric constant in the semiconductor ($\varepsilon_s = 11.8$).

The present formulation takes into account the "tail" of the density distribution of the majority charge carriers in the transitional region between the space-charge region and the quasineutral volume of the semiconductor [3], which is described by the exponential term in the Poisson equation, and which is usually discarded in the approximation of a Schottky barrier [1]. The capacitance C of the space charge, normalized over the contact area and dependent on the bias voltage v, was measured in the experiment. The concentration of the ionized impurity $\overline{N}$ and the width w of the depleted region are determined from the well-known formulas in terms of $C(v)$ [1]. However, $\overline{N}(w)$ corresponds to the true impurity distribution $N_n = N/n_0$ [$n_0 = n_i \exp(-\beta\,\varphi_0)$] only for smooth variations of N_n, i.e., those satisfying the condition $dN_n/dx \ll N_n/L_d$ (L_d is the Debye length). When the case of sharp gradients of impurity concentration, most often encountered in practice, occurs, $\overline{N}$ differs considerably from N_n. The problem of determining the $N(x)$ profile in the case of large gradients in the concentration of the impurity and with allowance for the majority charge carriers in the space-charge region is important in this connection.

The direct problem of finding the variation of the electrostatic potential with depth for a specified concentration distribution of an ionized impurity has been studied in many papers [1–3], where Eq. (1),

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considered for $x > 0$, was supplemented by the boundary conditions

$$\int_0^\infty \frac{d\Psi}{dx}\,dx = v, \qquad \left(\frac{d\Psi}{dx}, \Psi\right) \to 0 \quad \text{for} \quad x \to \infty \tag{3}$$

(v is a specified parameter).

In the present work, we solve the inverse problem of finding the concentration distribution of an ionized impurity with depth. We assume that, under quasiequilibrium conditions, the electrochemical potential φ_0 does not depend on depth and, in addition to (3), the following conditions are satisfied:

$$\int_0^\infty \left(\frac{d\Psi}{dx}\right)^2 dx = J_0(v), \qquad \int_0^\infty \frac{d^2\Psi}{dx^2}\,dx = z_0(v); \tag{4}$$

$$\frac{N(x)}{n_0} > 0, \qquad (N(x) - n_0) \to 0 \quad \text{for} \quad x \to \infty, \tag{5}$$

where $J_0(v)$ and $z_0(v)$ are functions determined from experiment by measuring the volt–farad characteristics of the depleted layer. In addition to (5), we assume that $N(x)$ is a two-parameter step function of the type

$$\frac{N(x)}{n_0} = \begin{cases} a, & 0 \leqslant x < \rho, \\ 1, & x \geqslant \rho, \end{cases} \tag{6}$$

where a and ρ are positive constants to be determined.

Statement of the Problem. We set

$$u = \beta\Psi, \qquad \xi = \frac{x}{|L|}, \qquad L^2 = \frac{\varepsilon_0\varepsilon_s kT}{2q^2 n_i}, \qquad q \neq 0.$$

In the new variables, conditions (1)–(5) take the form

$$2u'' = \exp(u) - \frac{N(\xi)}{n_0}, \quad u' \equiv \frac{du}{d\xi}; \tag{7}$$

$$(u, u', N - n_0) \to 0 \quad \text{for} \quad \xi \to \infty; \tag{8}$$

$$u(0) = -\beta v \equiv u_0; \tag{9}$$

$$u'(0) = -\beta|L|z_0 \equiv -z; \tag{10}$$

$$\int_0^\infty u'^2\,d\xi = \beta^2|L|J_0 \equiv J. \tag{11}$$

By virtue of (6), the second derivative u'' can have a discontinuity of the first kind. The first derivative u' and, hence, the function $u(\xi)$ itself are continuous.

Assuming the existence of the sought parameters $a > 0$ and $\rho > 0$, we consider Eq. (7) for $\xi > \rho$: $2u'' = e^u - 1$. With allowance for the first two conditions, from (8) we derive

$$u'^2 = e^u - u - 1 \equiv g_\infty(u), \tag{12}$$

where $g_\infty(u) \geqslant 0$ for all $u \in (-\infty, \infty)$. Similarly, from the equation $2u'' = e^u - a$ for $\xi < \rho$ we obtain $u'^2 + au - e^u = C$. The constant C is determined from the condition of continuity of $u(\xi)$ and $u'(\xi)$ at the point $\xi = \rho$:

$$C = (a-1)u_\rho - 1, \qquad u_\rho \equiv u(\rho).$$

For all $\xi \leqslant \rho$, we therefore have

$$u'^2 = e^u - au + (a-1)u_\rho - 1 \equiv g_a(u). \tag{13}$$

The conditions under which $g_a(u)$ is guaranteed to be nonnegative are laid out in Lemma 1. In extracting the roots in equalities (12) and (13), the required sign is determined by boundary conditions (9)–(11). The following versions are possible:

$$\text{(A)} \quad u' \geqslant 0, \quad u(0) < 0;$$

$$\text{(B)} \quad u' \leqslant 0, \quad u(0) > 0.$$

The case $u(0) = 0$ leads to a trivial solution. Here and below, therefore, we have $u(0) \neq 0$. In this paper, we consider only version (A), corresponding to depletion of the space-charge region.

Problem A. Let the condition (A) hold. We must determine the continuously differentiable function $u(\xi)$ and the positive parameters a and ρ satisfying the equations

$$u' = \sqrt{g_\infty(u)}, \quad \xi \geqslant \rho, \quad (u, u') \to 0 \quad \text{for} \quad \xi \to \infty;$$

$$u' = \sqrt{g_a(u)}, \quad 0 \leqslant \xi \leqslant \rho, \quad u(0) = u_0 < 0, \quad u'(0) = -z \geqslant 0, \quad \int_0^\infty u'^2 \, d\xi = J,$$

where u_0, z, and J are given real parameters.

The initial data u_0, z, and J are assumed to be independent in general. It is convenient to describe the structure of the set of values of u_0 and z for which $g_a(u) \geqslant 0$, $a > 0$, and $\rho > 0$ using the parameter

$$F(u_0, z) = \frac{1}{u_0}(g_\infty(u_0) + u_0 - z^2). \tag{14}$$

By virtue of (13) with $\xi = 0$ and (14), we have

$$u_0(1 - F) = (a - 1)(u_\rho - u_0). \tag{15}$$

The case of $F = 1$ ultimately leads to the equality $a = 1$ and reduces problem (A) to the form

$$u' = \sqrt{g_\infty(u)}, \quad \xi > 0, \quad u(0) = u_0, \quad u'(0) = -z,$$

$$\int_0^\infty u'^2 d\xi = J, \quad (u, u') \to 0 \quad \text{for} \quad \xi \to \infty.$$

This problem can be solved when there is certain conformity in the initial data. Below, we assume that $F \neq 1$ ($a \neq 1$). We can thus represent the unknown parameter u_ρ from (15) in the form

$$u_\rho = u_0 \frac{a - F}{a - 1}. \tag{16}$$

Solvability of Problem (A). We shall obtain a solution of problem (A) in quadratures. But first we must specify the conditions for nonnegativity of $g_a(u)$.

Because $u(\xi)$ is monotonic, we have the inequalities

$$u_0 \leqslant u(\xi) \leqslant u_\rho, \quad 0 \leqslant \xi \leqslant \rho; \qquad u_\rho \leqslant u(\xi) \leqslant 0, \quad \xi \geqslant \rho.$$

From these inequalities and (16), we find that $a \geqslant F > 1$ for $F > 1$. For all $u \in [u_0, u_\rho]$, therefore, we have

$$g_a(u) = g_\infty(u) + (a - 1)(u_\rho - u) \geqslant 0.$$

Since, under the conditions of problem A, we have

$$F = \frac{1 - \exp(u_0) + z^2}{|u_0|} > 0,$$

we assume that $F < 1$. Then, by analogy with the foregoing, we obtain $a \leqslant F < 1$. If $a < F$ and, in addition,

$$\exp(u_\rho) \leqslant a, \tag{17}$$

we have $0 < g_\infty(u_\rho) \leqslant g_a(u) \leqslant z^2$. Inequality (17) is equivalent to the condition of nonnegativity of the function

$$w(a) = |u_0|\frac{F-a}{1-a} + \ln a, \quad a \in (0, F). \tag{18}$$

It is easy to see that for $a \in (0, F)$ we have

$$\max w(a) = w(a_*) = |u_0| - S(\tau),$$

where

$$a_* = 1 + \frac{1}{2}\eta - \sqrt{\eta + \frac{1}{4}\eta^2}, \qquad a_* \in (0, F), \qquad S(\tau) = \tau + \ln(1+\tau),$$

$$\tau = \frac{1}{2} + \sqrt{\eta + \frac{1}{4}\eta^2} > 0, \qquad \eta = |u_0|(1-F) = g_\infty(u_0) - z^2 > 0.$$

Since the function $S(y)$ increases monotonically for $y \geqslant 0$, the equation

$$|u_0| = S(y) \tag{19}$$

has a unique solution $y_*(|u_0|)$ for each finite $u_0 \neq 0$. For $F < 1$, therefore, we can always indicate values of u_0 and z for which $\max w(a) > 0$. For these values of the initial data, the function (18) has two zeroes, a_1^0 and a_2^0, with $0 < a_1^0 < a_* < a_2^0 < F$. The function $g_a(u)$ is thus positive for all $a \in (a_1^0, a_2^0)$ and $u \in [u_0, u_\rho]$.

Lemma 1. *Let $u_0 < 0$ and $z < 0$ be initial data of problem* A.

1. *If $F = 1$, then $a = 1$ and $g_a(u) = g_\infty(u) \geqslant 0$.*
2. *If $F > 1$, then $a \geqslant F$ and $g_a(u) \geqslant 0$.*
3. *If $F < 1$ and $g_\infty(u_0) - z^2 < y_*/(1+y_*^2)$* [$y_*$ is a root of Eq. (19)], *then $0 < a_1^0 < a < a_2^0 < F$ and $g_a(u) > 0$, where a_1^0 and a_2^0 are roots of the equation*

$$|u_0|\frac{F-a}{1-a} + \ln a = 0. \tag{20}$$

With allowance for this lemma, the proof that problem A is solvable employs the following scheme. Let u_0, z, and J be given. By calculating values of F, we determine the set of values of the unknown parameter a and the parameter u_ρ. The function $g_a(u)$ is definite and nonnegative in this set. Using the inequalities

$$\frac{1}{2}e^{u_0}u^2(\xi) < \frac{1}{2}e^{u_\rho}u^2(\xi) \leqslant g_\infty(u) \leqslant \frac{1}{2}u^2(\xi), \qquad u(\xi) \in [u_0, 0]$$

for all possible values of a except for $a = F$, we have

$$0 < \frac{1}{2}e^{u_0}u_\rho^2 \leqslant g_\infty(u_\rho) \leqslant g_a(u) \leqslant z^2. \tag{21}$$

By virtue of (21), the solution of the problem

$$u' = \sqrt{g_a(u)}, \qquad 0 < \xi < \rho, \qquad u(0) = u_0 \tag{22}$$

can be represented in the form

$$K(u(\xi), a, u_0, z) \equiv \int_{u_0}^{u(\xi)} g_a^{-1/2}(s)\, ds = \xi. \tag{23}$$

From this, using the condition $u(\rho) = u_\rho$, we have

$$\rho = K(u_\rho, a, u_0, z), \quad u(\xi) = \tilde{U}(\xi, a, u_0, z), \quad \xi \in [0, \rho], \tag{24}$$

where $\tilde{U}(\xi, a, u_0, z)$ is a function inverse to $K(u(\xi), a, u_0, z)$. Similarly, for $\xi \geqslant \rho$, the solution of the problem

$$u' = \sqrt{g_\infty(u)}, \quad u(\rho) = u_\rho \tag{25}$$

has the form

$$K_\infty(u(\xi), a, u_0, z) \equiv \int\limits_{u_\rho}^{u(\xi)} g_\infty^{-1/2}(s)\, ds = \xi - \rho, \qquad u(\xi) = U_\infty(\xi, a, u_0, z).$$

For all $\xi \geqslant 0$, we thus have

$$u(\xi) = U(\xi, a, u_0, z), \qquad \rho = \rho(a, u_0, z), \tag{26}$$

and, hence, the functional

$$N = \int\limits_0^\infty u'^2\, d\xi = \int\limits_0^\rho \left|\frac{d\tilde{U}}{d\xi}\right|^2 d\xi + \int\limits_\rho^\infty \left|\frac{dU_\infty}{d\xi}\right|^2 d\xi \tag{27}$$

depends on a, u_0, and z. Fixing the values of u_0 and z and employing the latter condition in the formulation of problem A, we arrive at the equation

$$N(a) = J. \tag{28}$$

Solutions of the latter yield the values of the parameter a being sought. After a is found, the parameter u_ρ is determined from (16), the parameter ρ from (24), and the unknown function $u(\xi)$ from (26). The question of the solvability of problem A thus comes down to an analysis of the solvability of Eq. (28). Let us investigate the properties of the function $N(a)$.

Let $u_1(\xi)$, ρ_1, and $u_2(\xi)$, ρ_2 be solutions of problems (22) and (25) that correspond to the values a_1 and a_2 of the parameter a. We set $\hat{u}(\xi) = u_1(\xi) - u_2(\xi)$, $\hat{\rho} = \rho_1 - \rho_2$, and $\hat{a} = a_1 - a_2$, and with no loss of generality we take $\hat{a} < 0$. We have

$$\hat{u}_\rho \equiv u_{\rho_1} - u_{\rho_2} = \hat{a}\frac{u_0(F-1)}{(a_1-1)(a_2-1)}. \tag{29}$$

Lemma 2. *Let a_1 and a_2 be arbitrary values of the parameter a that simultaneously satisfy conditions 2 or 3 of Lemma 1. Let $a_1 < a_2$. Then $\hat{u}(\xi) \geqslant 0$ for all $\xi \geqslant 0$, $\hat{\rho} > 0$ for $F > 1$, and $\hat{\rho} < 0$ for $F < 1$. Then $N(a)$ of (28) is a monotonically decreasing function.*

Proof. We consider two variants: $F > 1$ and $F \in (0,1)$. For $a_1 > F > 1$, from (29) we have $\hat{u}_\rho > 0$. Since we have $g_{a_1}(s) - g_{a_2}(s) = \hat{a}(u_0 - s)$, for $s > u_0$ and for $\xi \in (0, \min(\rho_1, \rho_2)]$ we get $\hat{u}(\xi) > 0$ from (23). In this interval we have

$$\int\limits_{u_2(\xi)}^{u_1(\xi)} g_a^{-1/2}(s)\, ds = -\hat{a} \int\limits_{u_0}^{u_2(\xi)} f(s)\, ds, \tag{30}$$

where

$$f(s) = \frac{(s-u_0)}{(g_{a_1}^{1/2}(s) + g_{a_2}^{1/2}(s))g_{a_1}^{1/2}(s)g_{a_2}^{1/2}(s)}.$$

We take $\hat{\rho} < 0$. Because $u_i(\xi)$ are monotonic over the interval $[\rho_1, \rho_2]$, we have

$$u_i(\rho_1) \leqslant u_i(\xi) \leqslant u_i(\rho_2), \quad i = 1, 2.$$

Hence we have $0 < \hat{u}_\rho \leqslant \hat{u}(\xi) \leqslant u_1(\rho_2) - u_2(\rho_1)$ and, in particular, $\hat{u}(\rho_2) > 0$. For all $\xi \geqslant \rho_2$, therefore, from the representation

$$\xi - \rho_2 = \int\limits_{u_1(\rho_2)}^{u_1(\xi)} g_\infty^{-1/2}(s)\, ds = \int\limits_{u(\rho_2)}^{u_2(\xi)} g_\infty^{-1/2}(s)\, ds \tag{31}$$

we get $\hat{u}(\xi) \geqslant 0$. If $\hat{\rho} > 0$, then in the interval $[\rho_2, \rho_1]$, from the representation

$$\xi - \rho_2 = \int\limits_{u_1(\rho_2)}^{u_1(\xi)} g_{a_1}^{-1/2}(s)\, ds = \int\limits_{u_2(\rho_2)}^{u_2(\xi)} g_{\infty}^{-1/2}(s)\, ds$$

with allowance for the inequality $g_{a_1}(s) > g_{\infty}(s)$, as well as the inequality $\hat{u}(\rho_2) > 0$ obtained earlier, we have

$$\int\limits_{u_1(\rho_2)}^{u_1(\xi)} g_{\infty}^{-1/2}(s)\, ds > \int\limits_{u_1(\rho_2)}^{u_1(\xi)} g_{a_1}^{-1/2}(s)\, ds > \int\limits_{u_1(\rho_2)}^{u_2(\xi)} g_{\infty}^{-1/2}(s)\, ds.$$

From this we have $\hat{u}(\xi) > 0$ for $\xi \in [\rho_2, \rho_1]$. Using (31) with ρ_2 replaced by ρ_1, we obtain $\hat{u}(\xi) \geqslant 0$ for all $\xi \geqslant \rho_1$. We thus have $\hat{u}(\xi) \geqslant 0$ for all $\xi \in (0, \infty)$ in the case of $a_1 < a_2$. We multiply Eq. (7) by the sufficiently smooth function $\psi(\xi)$ and integrate the resulting equation over ξ from zero to some $R > \rho$. Taking R to infinity, with allowance for the boundary values (8)–(11) we obtain the equality

$$\int\limits_0^{\infty} (2u'\psi' + \psi(\mathrm{e}^u - 1))\, d\xi + (1 - a) \int\limits_0^{\rho} \psi\, d\xi = -2u'(0)\psi(0). \tag{32}$$

For $\hat{u}(\xi)$, from (32) we have

$$\int\limits_0^{\infty} (2\hat{u}'\psi' + \psi(\mathrm{e}^{u_1} - \mathrm{e}^{u_2}))\, d\xi - \hat{a} \int\limits_0^{\rho_1} \psi\, d\xi = -(a_2 - 1) \int\limits_{\rho_1}^{\rho_2} \psi\, d\xi. \tag{33}$$

For $\psi = 1$, from (33) we derive

$$\int\limits_0^{\infty} (\mathrm{e}^{u_1} - \mathrm{e}^{u_2})\, d\xi + |\hat{a}|\rho_1 = (a_2 - 1)\hat{\rho}, \tag{34}$$

and, hence, $\hat{\rho} > 0$. We set

$$N(a_i) = \int\limits_0^{\infty} \left(\frac{du_i}{dx}\right)^2 d\xi.$$

With allowance for (34) and the condition $\hat{u}(0) = 0$, we have

$$N(a_1) - N(a_2) = -\int\limits_0^{\rho_2} (\mathrm{e}^{u_1} - a_1 + \mathrm{e}^{u_2} - a_2)\hat{u}\, d\xi - \int\limits_{\rho_2}^{\rho_1} (\mathrm{e}^{u_1} - a_1 + \mathrm{e}^{u_2} - 1)\hat{u}\, d\xi - \int\limits_{\rho_1}^{\infty} (\mathrm{e}^{u_1} - 1 + \mathrm{e}^{u_2} - 1)\hat{u}\, d\xi. \tag{35}$$

In accordance with the foregoing, the right side of (35) is positive. Therefore $N(a)$ for $F > 1$ decreases monotonically from $N_F = N(F)$ to $N_{\infty} = N(a_0)$, where $a_0 = \sup a$.

Let $F < 1$ and let condition 3 of Lemma 1 be satisfied, i.e., $0 < a_1^0 < a_1 < a_2 < a_2^0 < F$ and $\exp(u_i) \leqslant a_i$. From (29) we get $\hat{u}_{\rho} < 0$. In this case $\hat{\rho} < 0$, since

$$\rho_2 = \int\limits_{u_0}^{u_{\rho_2}} g_{a_2}^{-1/2}(s)\, ds = \int\limits_{u_0}^{u_{\rho_1}} g_{a_2}^{-1/2}(s)\, ds + \int\limits_{u_{\rho_1}}^{u_{\rho_2}} g_{a_2}^{-1/2}(s)\, ds > \rho_1.$$

From (30) we derive $\hat{u}(\xi) > 0$ for $\xi \in (0, \rho_1]$. For $\xi \in [\rho_1, \rho_2]$, with allowance for the inequality $\hat{u}(\rho_1) > 0$, we obtain

$$\xi - \rho_1 = \int\limits_{u_1(\rho_1)}^{u_1(\xi)} g_{\infty}^{-1/2}(s)\, ds = \int\limits_{u_2(\rho_1)}^{u_1(\rho_1)} g_{a_2}^{-1/2}(s)\, ds + \int\limits_{u_1(\rho_1)}^{u_2(\xi)} g_{a_2}^{-1/2}(s)\, ds > \int\limits_{u_1(\rho_1)}^{u_2(\xi)} g_{\infty}^{-1/2}(s)\, ds.$$

Hence, we have $\hat{u}(\xi) > 0$ and, in particular, $\hat{u}(\rho_2) > 0$. Using (31), we obtain $\hat{u}(\xi) > 0$ for $\xi > 0$. By virtue of (35), the function $N(a)$ for $F < 1$ decreases monotonically over the interval (a_1^0, a_2^0).

Remark 1. Let $F > 1$ and $a = F$ ($u_\rho = 0$). From (25) we get $u(\xi) = u'(\xi) = 0$ for $\xi \geqslant \rho$. Problems (22) and (25) reduce to the problem

$$u' = \sqrt{g_F(u)}, \quad 0 < \xi < \rho, \quad u(0) = u_0; \quad u(\xi) = 0, \quad \xi \geqslant \rho.$$

The solution $u_F(\xi)$ and ρ_F of this problem is given by Eqs. (23) and (24). We have therefore determined the functional

$$N(a)|_{a=F} \equiv N_F = \int_0^{\rho_F} \left(\frac{du_F}{dx}\right)^2 d\xi. \tag{36}$$

Remark 2. In equality (32) we set $\psi(\xi) = u(\xi)$ and we use the last condition of problem (A). We obtain

$$\int_0^\infty u(e^u - 1)\, d\xi + (1 - a)\int_0^\rho u\, d\xi = -2(u'(0)u(0) + J). \tag{37}$$

For $F > 1$, the left side of (37) is nonnegative. We arrive at the necessary condition for problem (A) to be solvable for $F > 1$:

$$0 < J < -u(0)u'(0).$$

Remark 3. In formulas (16), (25), and (27) we go to the limit as $a \to \infty$. We have

$$\lim_{a\to\infty} u_\rho = u_0, \quad \lim_{a\to\infty} g_\infty(u_\rho) = g_\infty(u_0), \quad \lim_{a\to\infty} \rho = 0. \tag{38}$$

The last equality is a consequence of Lemma 2 for $F > 1$ and the inequality

$$\rho \leqslant \int_{u_0}^{u_\rho} ((a-1)(u_\rho - s))^{-1/2}\, ds = \frac{2}{a-1}\sqrt{|u_0|(F-1)}.$$

It is easy to show that we have

$$\lim_{a\to\infty} \rho(a-1) = 2(|z| - \sqrt{g_\infty(u_0)}). \tag{39}$$

Since $\rho \to 0$, Eq. (22) loses meaning [the limiting value of the function $(a-1)(u_\rho - u(\xi))$ and, hence, the derivative $u'(\xi)$ are undefined]. The solution $u_\infty(\xi)$ of problem (25) as $a \to \infty$ is defined, however, and it can be represented in the form

$$\xi = \int_{u_0}^{u_\infty(\xi)} g_\infty^{-1/2}(s)\, ds, \qquad u_\infty(0) = u_0.$$

For the limiting value of N from (28), using (38), (39), and the mean value theorem, we obtain

$$N_\infty = \lim_{a\to\infty} N = \int_0^\infty g_\infty(u_0)\, d\xi. \tag{40}$$

Theorem 1. *A unique solution $(u(\xi), a, \rho)$ of problem* (A) *exists if, in addition to the assumptions of Lemma* 1, *the following conditions on the initial data are satisfied:*
if $F > 1$, then

$$N_\infty < J \leqslant \min(N_F, |u_0 z|); \tag{41}$$

if $F < 1$, then

$$N(a_2^0) < J < N(a_1^0), \tag{42}$$

where N_F, N_∞, and $N(a)$ are defined by formulas (27), (36), *and* (40), *and a_1^0 and a_2^0 and are roots of Eq.* (20).

Proof. Because the function $N(a)$ is monotonic, conditions (41) and (42) ensure the existence of a unique root a^* of Eq. (28). From (16) the parameter u_ρ is uniquely determined for a^* and the parameter ρ is uniquely determined from (24). For the given a^*, u_ρ, and ρ, the function $u(\xi)$ is uniquely determined from formulas (23) and (24).

A unique solution of problem A thus exists. This result can have great practical importance in the determination of the concentration profile of an ionized impurity from the data of C–V measurements on metal–semiconductor structures in the case of large depth gradients of the impurity concentration.

REFERENCES

1. S. H. Sze, *Physics of Semiconductor Devices*, John Wiley and Sons, New York (1981).
2. V. I. Gaman, *Physics of Semiconductor Devices* [in Russian], Izd. Tomsk Univ., Tomsk (1989).
3. R. W. Klopfenstein and C. P. Wu, "Computer solution on one-dimensional Poisson's equation," *IEEE Trans. Electron Devices*, **ED-22**, No. 6, 329–333 (1975).